Factor Model Based Risk Management for Hedge Funds and Fund of Hedge Funds with Limited Transparency

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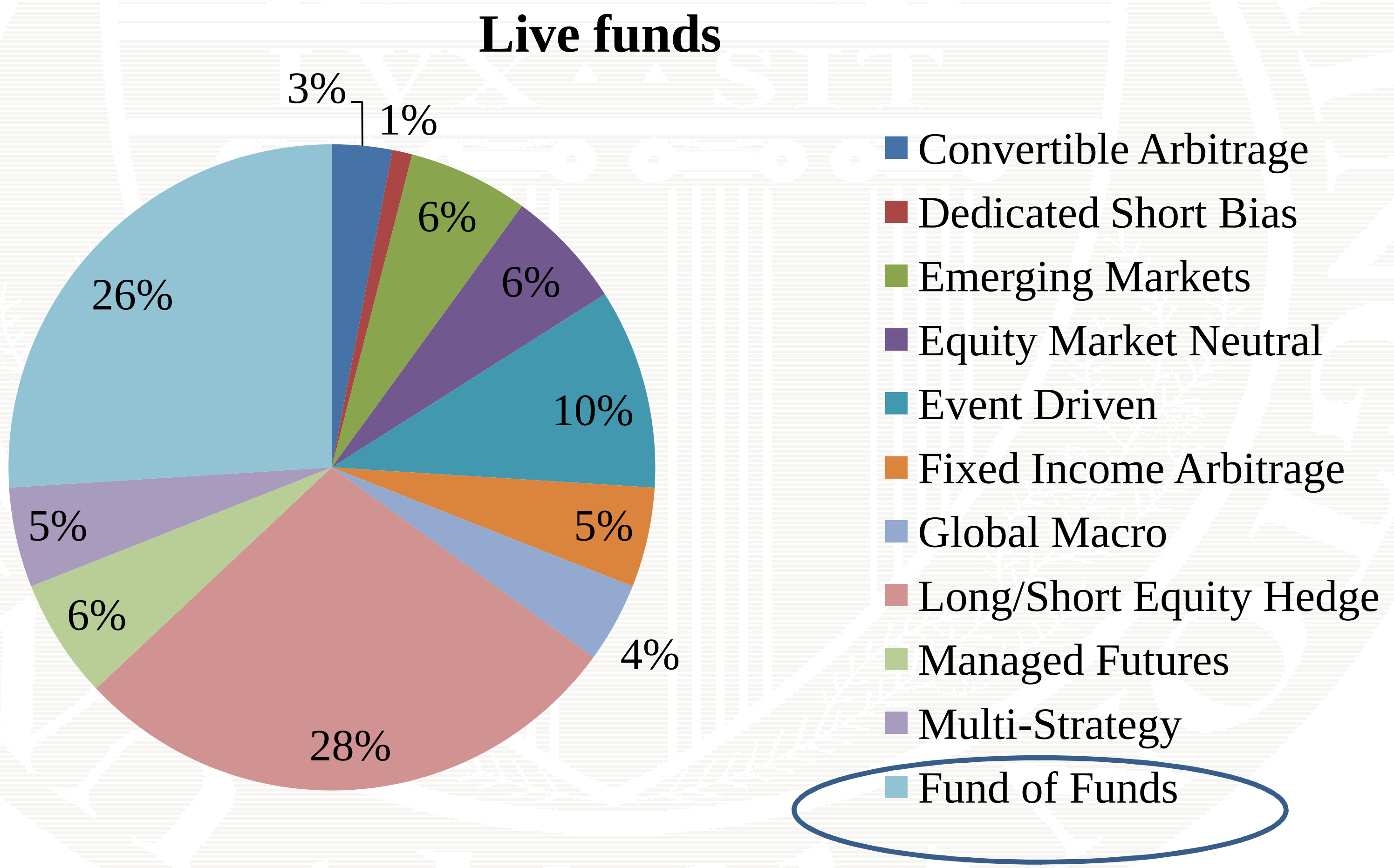
Outline

- Fund of hedge funds environment
- Characteristics of hedge fund data
- Linear factor model for hedge fund returns
- Risk Measures
- Factor model Monte Carlo methodology
- Risk decompositions
- Example

Fund of Hedge Funds Environment

- FoHFs are hedge funds that invest in other hedge funds
  - 20 to 30 portfolios of hedge funds
  - Typical portfolio size is 30 funds
- Hedge fund universe is large: 5000 live funds
  - Segmented into 10-15 distinct strategy types
- Hedge funds voluntarily report monthly performance to commercial databases
  - Altvest, CISDM, HedgeFund.net, Lipper TASS, CS/Tremont, HFR
- Limited transparency is typical but that is changing

Hedge Fund Universe



Risk Measurement and Management

- Quantify exposures to risk drivers
  - Equity, rates, credit, volatility, currency, commodity, strategy, etc.
- Quantify fund and portfolio risk
  - Return standard deviation, value-at-risk (VaR), expected tail loss (ETL)
- Perform risk decomposition
  - Contribution of risk factors, contribution of constituent funds to portfolio risk
- Stress testing and scenario analysis

Challenges of Hedge Fund Return Data

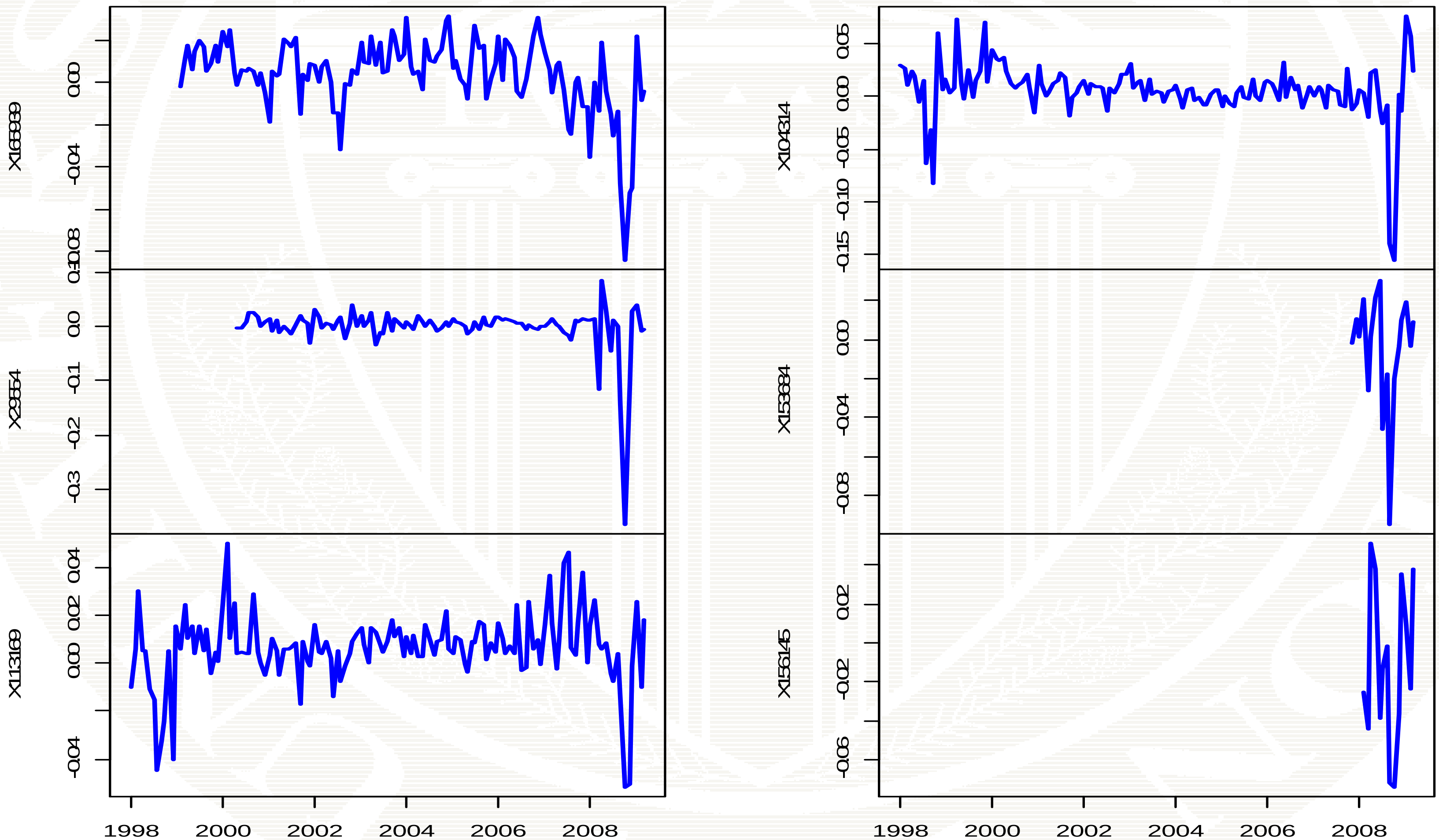
- Reporting biases
  - Survivorship, backfill
- Non-normal behavior
  - Asymmetry (skewness) and fat tails (excess kurtosis)
- Serial correlation
  - Performance smoothing, illiquid positions
- Small sample sizes
- Unequal histories

Hedge Fund Data: Characteristics

	fund1	fund2	fund3	fund4	fund5
Observations	122.0000	107.0000	135.0000	135.0000	135.0000
NAs	13.0000	28.0000	0.0000	0.0000	0.0000
Minimum	-0.0842	-0.3649	-0.0519	-0.1556	-0.2900
Quartile 1	-0.0016	-0.0051	0.0020	-0.0017	-0.0021
Median	0.0058	0.0046	0.0060	0.0073	0.0049
Arithmetic Mean	0.0038	-0.0017	0.0063	0.0059	0.0021
Geometric Mean	0.0037	-0.0029	0.0062	0.0055	0.0014
Quartile 3	0.0158	0.0129	0.0127	0.0157	0.0127
Maximum	0.0311	0.0861	0.0502	0.0762	0.0877
Variance	0.0003	0.0020	0.0002	0.0008	0.0013
Stdev	0.0176	0.0443	0.0152	0.0275	0.0357
Skewness	-1.7753	-5.6202	-0.8810	-2.4839	-4.9948
Kurtosis	5.2887	40.9681	3.7960	13.8201	35.8623
Rho1	0.6060	0.3820	0.3590	0.4400	0.383

Sample: January 1998 – March 2009

Hedge Fund Data: Unequal Histories



Fund Level Linear Factor Model

$$R_{it} = \alpha_i + \beta_{i1}F_{1t} + \cdots + \beta_{ik}F_{kt} + \varepsilon_{it},$$

$$= \alpha_i + \boldsymbol{\beta}_i'\mathbf{F}_t + \varepsilon_{it}$$

$$i = 1, \ldots, n; \; t = t_i, \ldots, T$$

$$\mathbf{F}_t \sim (\boldsymbol{\mu}_F, \boldsymbol{\Sigma}_F)$$

$$\varepsilon_{it} \sim (0, \sigma^2_{\varepsilon,i})$$

$$\text{cov}(F_{jt}, \varepsilon_{is}) = 0 \text{ for all } j, \; i, \; s \text{ and } t$$

$$\text{cov}(\varepsilon_{it}, \varepsilon_{js}) = 0 \text{ for } i \neq j, s \text{ and } t$$

Performance Attribution

$$E[R_{it}] = \alpha_i + \beta_{i1}E[F_{1t}] + \cdots + \beta_{ik}E[F_{kt}]$$

Expected return due to systematic "beta" exposure

$$\beta_{i1}E[F_{1t}] + \cdots + \beta_{ik}E[F_{kt}]$$

Expected return due to manager specific "alpha"

$$\alpha_i = E[R_{it}] - (\beta_{i1}E[F_{1t}] + \cdots + \beta_{ik}E[F_{kt}])$$

Factor Model Covariance

$$\underset{n\times 1}{\mathbf{R}_t} = \underset{n\times 1}{\boldsymbol{\alpha}} + \underset{n\times k}{\mathbf{B}}\underset{k\times 1}{\mathbf{F}_t} + \underset{n\times 1}{\boldsymbol{\varepsilon}_t}$$

$$\text{var}(\mathbf{R}_t) = \boldsymbol{\Sigma}_{FM} = \mathbf{B}\boldsymbol{\Sigma}_{\mathbf{F}}\mathbf{B}' + \mathbf{D}_{\varepsilon}$$

$$\mathbf{D}_{\varepsilon} = diag(\sigma^2_{\varepsilon,1},\ldots,\sigma^2_{\varepsilon,n})$$

Note: $\text{cov}(R_{it}, R_{jt}) = \boldsymbol{\beta}_i' \text{var}(\mathbf{F}_t)\boldsymbol{\beta}_j = \boldsymbol{\beta}_i'\boldsymbol{\Sigma}_{\mathbf{F}}\boldsymbol{\beta}_j$

Portfolio Linear Factor Model

$$\mathbf{w} = (w_1, \ldots, w_n)' = \text{ portfolio weights}$$

$$\sum_{i=1}^{n} w_i = 1, \; w_i \geq 0 \text{ for } i = 1, \ldots, n$$

$$\begin{aligned}
R_{p,t} &= \mathbf{w}'\mathbf{R}_t = \mathbf{w}'\boldsymbol{\alpha} + \mathbf{w}'\mathbf{B}\mathbf{F}_t + \mathbf{w}'\boldsymbol{\varepsilon}_t \\
&= \sum_{i=1}^{n} w_i R_{it} = \sum_{i=1}^{n} w_i \alpha_i + \sum_{i=1}^{n} w_i \boldsymbol{\beta}_i'\mathbf{F}_t + \sum_{i=1}^{n} w_i \varepsilon_{it} \\
&= \alpha_p + \boldsymbol{\beta}_p'\mathbf{F}_t + \varepsilon_{p,t}
\end{aligned}$$

Factor Structure

- Traditional Market Factors
  – Tradable indices that measure broad asset class performance of equity, rates, credit, commodities and currency. Factor is excess return on index.
  – Tradable market factors that measure intra-asset class biases
    - Geography, Sector, Style, Capitalization, Credit Quality, Capital Structure, Yield Curve Shape
- Hedge Fund Specific ("Exotic") Factors
  – Tradable factors that capture performance of passive investment strategies utilized by hedge funds

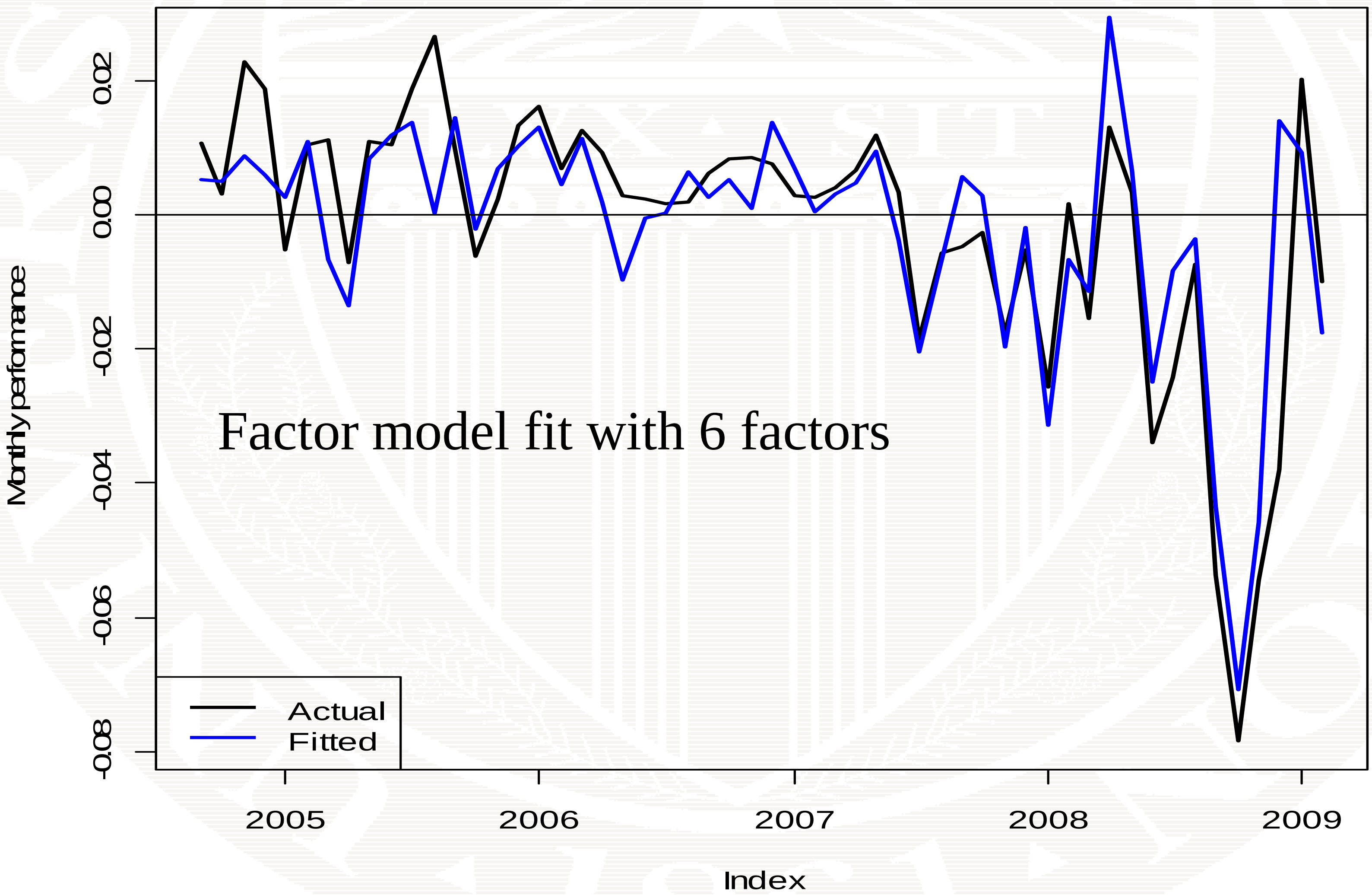
Practical Considerations

- Many potential risk factors (> 50)
- High collinearity among some factors
- Risk factors vary across discipline/strategy
- Nonlinear effects
- Dynamic (lagged) effects
- Factor sensitivities change over time
- Common histories for factors; unequal histories for fund performance

Estimation Methodology

- Choice of Factor Set
  - Statistical variable selection/Data mining techniques
  - Qualitative/subjective selection of factors
  - Hybrid approach
- Estimation technique
  - OLS on full sample
  - Rolling regression
  - Regression on exponentially weighted data
  - Kalman filter
- Use proxy models for funds with short history

Factor Model Fit: Example Fund



Risk Measures

Return Standard Deviation (*SD, aka active risk*)

$$\sigma = SD(R_t) = \left(\boldsymbol{\beta}'\boldsymbol{\Sigma}_F\boldsymbol{\beta} + \sigma_\varepsilon^2\right)^{1/2}$$

Value-at-Risk (*VaR*)

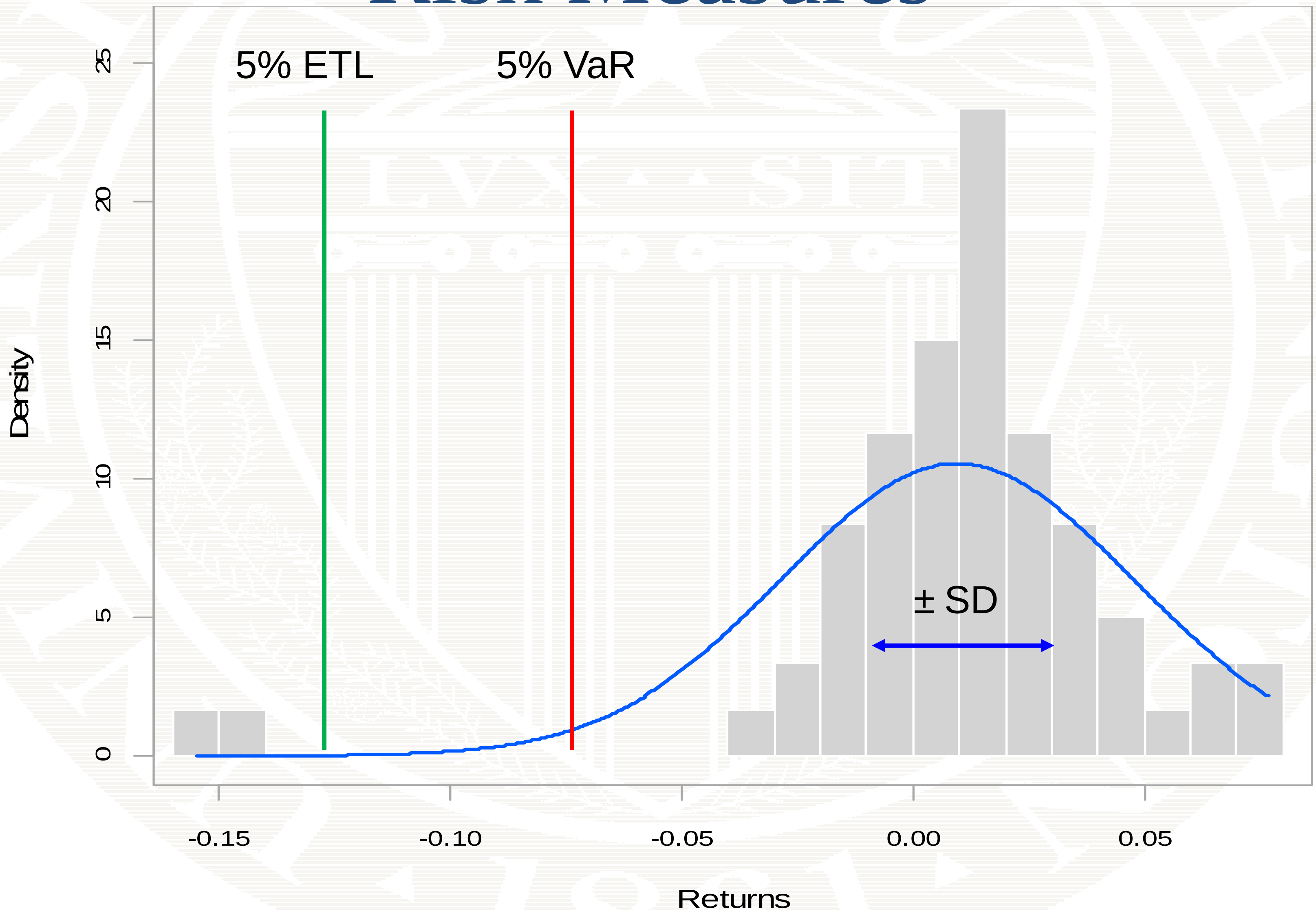
$$VaR_\alpha = q_\alpha = F^{-1}(\alpha),\ 0.01 \le \alpha \le 0.10$$

$$F = CDF \text{ of return } R_t$$

Expected Tail Loss (*ETL*)

$$ETL_\alpha = E[R_t \mid R_t \le VaR_\alpha]$$

Risk Measures



Tail Risk Measures: Non-Normal Distributions

- Hedge fund returns are typically non-normal
- Many possible univariate non-normal distributions
  - Student's-*t*, skewed-*t*, generalized hyperbolic, Gram-Charlier, α-stable, generalized Pareto, etc.
- Need multivariate non-normal distributions for portfolio analysis.
- Small samples and unequal histories make multivariate modeling difficult

Factor Model Monte Carlo (FMMC)

- Use fitted factor model to simulate pseudo hedge fund return data preserving empirical characteristics of risk factors and residuals
  - Use *full data* for factors and *unequal history* for funds to deal with missing data
- Estimate tail risk and related measures non-parametrically from simulated return data

FMMC: Unequal Histories

Risk factors

$F_{1,T}, \ldots, F_{k,T}$

$\vdots$

$F_{1,T-T_i}, \ldots, F_{k,T-T_i}$

$\vdots$

$F_{1,1}, \ldots, F_{k,1}$

Observe full history

Fund performance

$R_{1,T} \quad R_{n,T}$

$\vdots \quad \vdots$

$R_{1,T-T_1} \quad \vdots$

$R_{n,T-T_n}$

Observe partial histories

Simulation Algorithm

- Simulate *B* values of the risk factors by re-sampling from *full sample* empirical distribution:

$$\left\{\mathbf{F}_1^*, \ldots, \mathbf{F}_B^*\right\}$$

- Simulate *B* values of the factor model residuals from fitted non-normal distribution:

$$\left\{\hat{\varepsilon}_{i1}^*, \ldots, \hat{\varepsilon}_{iB}^*\right\}, \; i = 1, \ldots, n$$

- Create factor model returns from factor models fit over *truncated samples*, simulated factor variables drawn from *full sample* and simulated residuals:

$$R_{it}^* = \hat{\alpha}_i + \hat{\boldsymbol{\beta}}_i'\mathbf{F}_t^* + \hat{\varepsilon}_{it}^*, \; t = 1, \ldots, B; i = 1, \ldots, n$$

What to do with $\{R_{it}^*\}_{t=1}^{B}, \{\mathbf{F}_{it}^*\}_{t=1}^{B}, \{\hat{\varepsilon}_{it}^*\}_{t=1}^{B}$, ?

- Backfill missing fund performance
- Compute fund and portfolio performance measures (e.g., Sharpe ratios)
- Compute *non-parametric* estimates of fund and portfolio tail risk measures
- Compute *non-parametric* estimates of fund and factor contributions to portfolio tail risk measures

Factor Risk Budgeting

Given linear factor model for fund or portfolio returns,

$$R_t = \alpha + \boldsymbol{\beta}'\mathbf{F}_t + \varepsilon_t = \alpha + \boldsymbol{\beta}'\mathbf{F}_t + \sigma_\varepsilon \times z_t = \alpha + \tilde{\boldsymbol{\beta}}'\tilde{\mathbf{F}}_t$$

$$\tilde{\boldsymbol{\beta}}' = (\boldsymbol{\beta}', \sigma_\varepsilon),\ \tilde{\mathbf{F}}_t = (\mathbf{F}_t', z_t)',\ z_t \sim (0,1)$$

SD, *VaR* and *ETL* are linearly homogenous functions of factor sensitivities $\tilde{\boldsymbol{\beta}}$. Euler's theorem gives additive decomposition

$$RM(\tilde{\boldsymbol{\beta}}) = \sum_{j=1}^{k+1} \tilde{\beta}_j \frac{\partial RM(\tilde{\boldsymbol{\beta}})}{\partial \tilde{\beta}_j},\ RM = SD,\ VaR_\alpha,\ ETL_\alpha$$

Factor Contributions to Risk

Marginal Contribution to Risk of factor *j*: $$\frac{\partial RM(\tilde{\boldsymbol{\beta}})}{\partial \tilde{\beta}_j}$$

Contribution to Risk of factor *j*: $$\tilde{\beta}_j \frac{\partial RM(\tilde{\boldsymbol{\beta}})}{\partial \tilde{\beta}_j}$$

Percent Contribution to Risk of factor *j*: $$\tilde{\beta}_j \frac{\partial RM(\tilde{\boldsymbol{\beta}})}{\partial \tilde{\beta}_j} \bigg/ RM(\tilde{\boldsymbol{\beta}})$$

Factor Tail Risk Contributions

For *RM* = *VaR*, *ETL* it can be shown that

$$\frac{\partial VaR_{\alpha}(\tilde{\boldsymbol{\beta}})}{\partial \tilde{\beta}_j} = E[\tilde{F}_{jt} \mid R_t = VaR_{\alpha}],\ j = 1,\ldots,k+1$$

$$\frac{\partial ETL_{\alpha}(\tilde{\boldsymbol{\beta}})}{\partial \tilde{\beta}_j} = E[\tilde{F}_{jt} \mid R_t \leq VaR_{\alpha}],\ j = 1,\ldots,k+1$$

Notes:

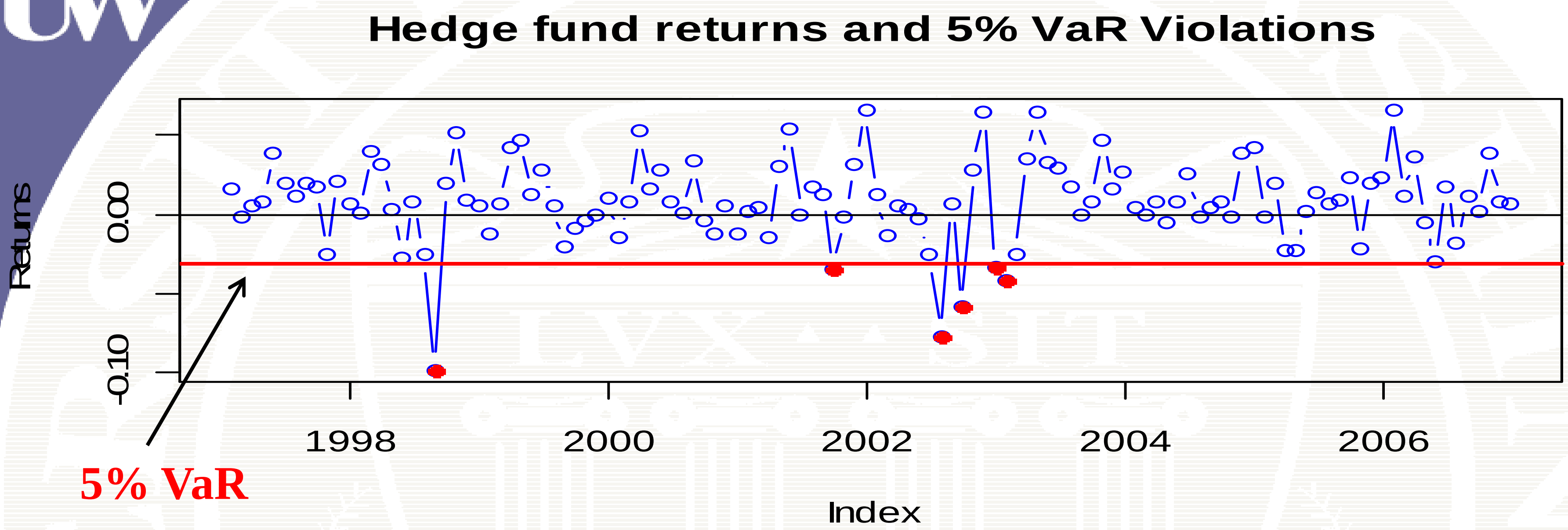
1. Intuitive interpretations as stress loss scenarios
2. Analytic results are available under normality

Semi-Parametric Estimation

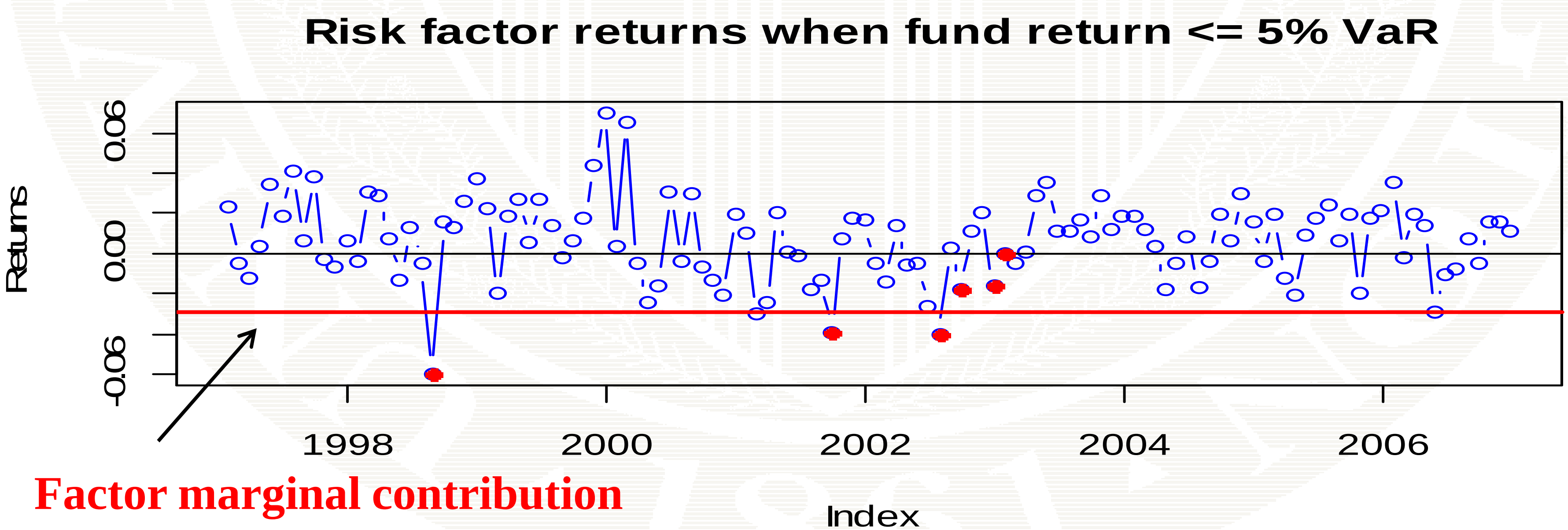
Factor Model Monte Carlo semi-parametric estimates

$$\hat{E}[\tilde{F}_{jt} \mid R_t = VaR_\alpha] = \frac{1}{m}\sum_{t=1}^{B} \tilde{F}_{jt}^{*} \cdot 1\left\{VaR_\alpha - \varepsilon \le R_t^{*} \le VaR_\alpha + \varepsilon\right\}$$

$$\hat{E}[\tilde{F}_{jt} \mid R_t \le VaR_\alpha] = \frac{1}{[B\alpha]}\sum_{t=1}^{B} \tilde{F}_{jt}^{*} \cdot 1\left\{R_t^{*} \le VaR_\alpha\right\}$$



5% VaR



Factor marginal contribution to 5% ETL

Portfolio Risk Budgeting

Given portfolio returns,

$$R_{p,t} = \mathbf{w}'\mathbf{R}_t = \sum_{i=1}^{n} w_i R_{it}$$

SD, *VaR* and *ETL* are linearly homogenous functions of portfolio weights **w**. Euler's theorem gives additive decomposition

$$RM(\mathbf{w}) = \sum_{i=1}^{n} w_i \frac{\partial RM(\mathbf{w})}{\partial w_i},\ RM = SD,\ VaR_\alpha,\ ETL_\alpha$$

Fund Contributions to Portfolio Risk

Marginal Contribution to Risk of fund *i*:

$$\frac{\partial RM(\mathbf{w})}{\partial w_i}$$

Contribution to Risk of fund *i*:

$$w_i \frac{\partial RM(\mathbf{w})}{\partial w_i}$$

Percent Contribution to Risk of fund *i*:

$$w_i \frac{\partial RM(\mathbf{w})}{\partial w_i} \Big/ RM(\mathbf{w})$$

Portfolio Tail Risk Contributions

For $RM = VaR$, ETL it can be shown that

$$\frac{\partial VaR_{\alpha}(\mathbf{w})}{\partial w_i} = E[R_{it} \mid R_{p,t} = VaR_{\alpha}(\mathbf{w})],\ i = 1,\ldots,n$$

$$\frac{\partial ETL_{\alpha}(\mathbf{w})}{\partial w_i} = E[R_{it} \mid R_{p,t} \leq VaR_{\alpha}(\mathbf{w})],\ i = 1,\ldots,n$$

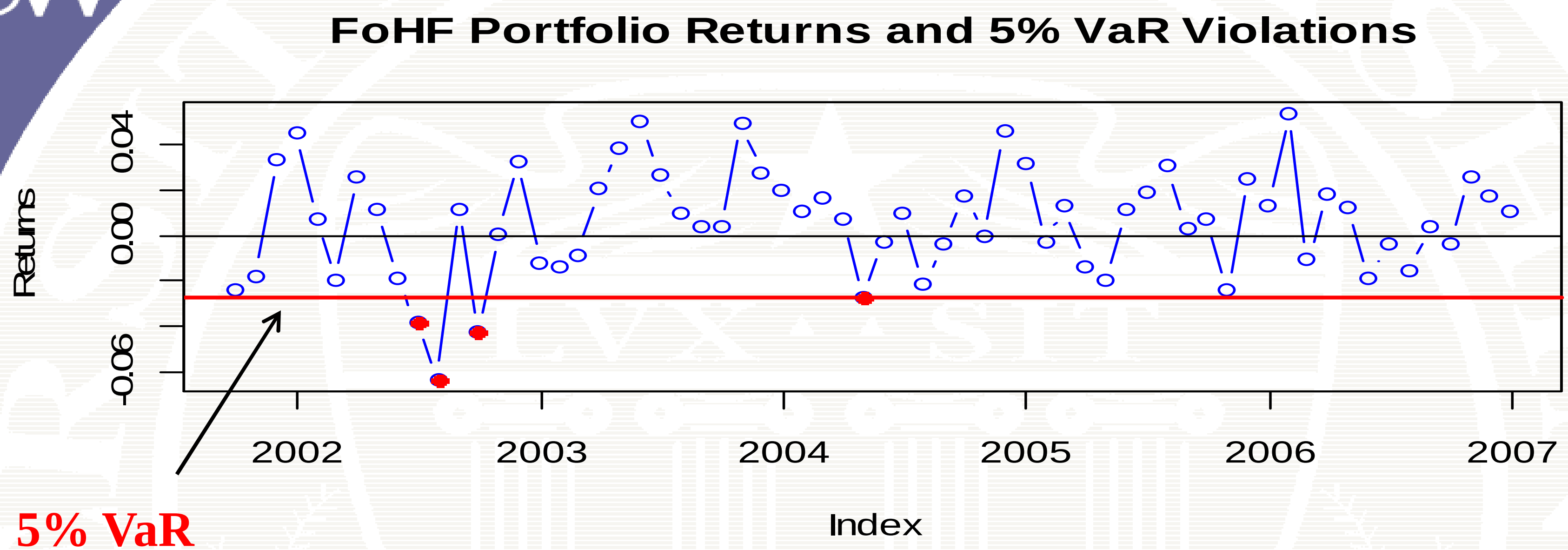
Note: Analytic results are available under normality

Semi-Parametric Estimation

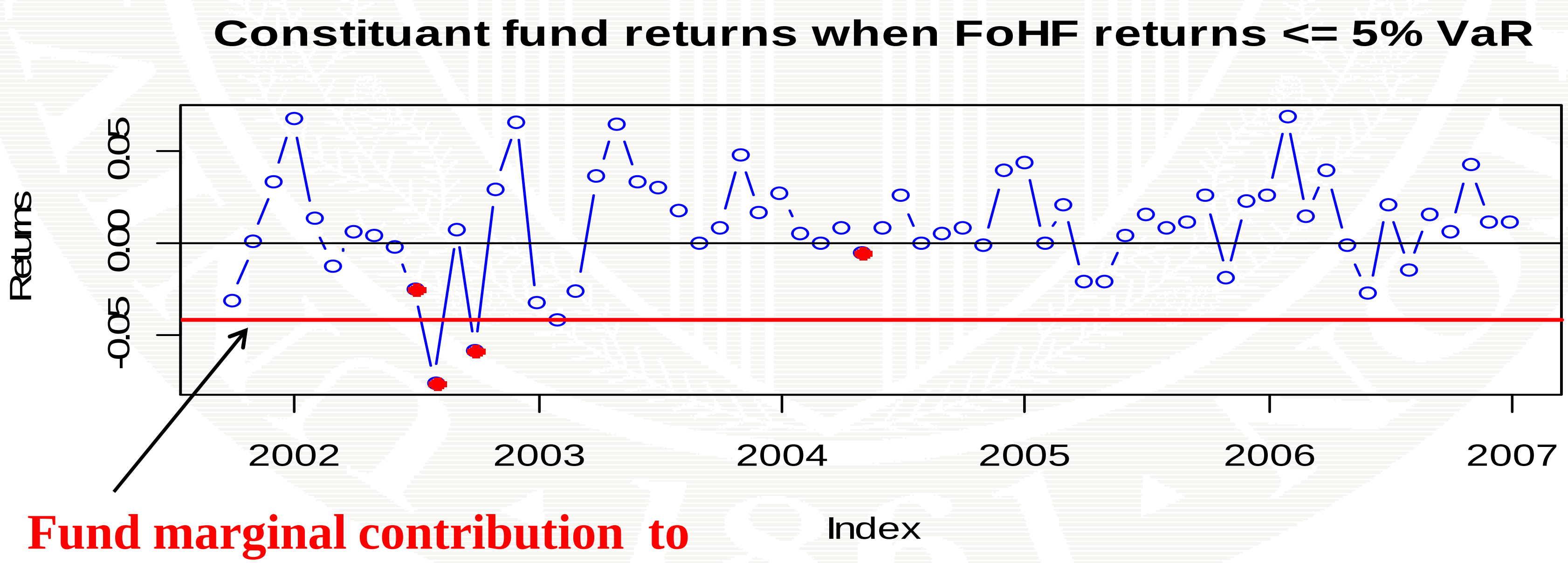
Factor Model Monte Carlo semi-parametric estimates

$$\hat{E}[R_{it} \mid R_{p,t} = VaR_{\alpha}(\mathbf{w})] = \frac{1}{m}\sum_{t=1}^{B} R_{it}^{*} \cdot 1\left\{VaR_{\alpha}(\mathbf{w}) - \varepsilon \leq R_{p,t}^{*} \leq VaR_{\alpha}(\mathbf{w}) + \varepsilon\right\}$$

$$\hat{E}[R_{it} \mid R_{t} \leq VaR_{\alpha}(\mathbf{w})] = \frac{1}{[B\alpha]}\sum_{t=1}^{B} R_{it}^{*} \cdot 1\left\{R_{p,t}^{*} \leq VaR_{\alpha}(\mathbf{w})\right\}$$



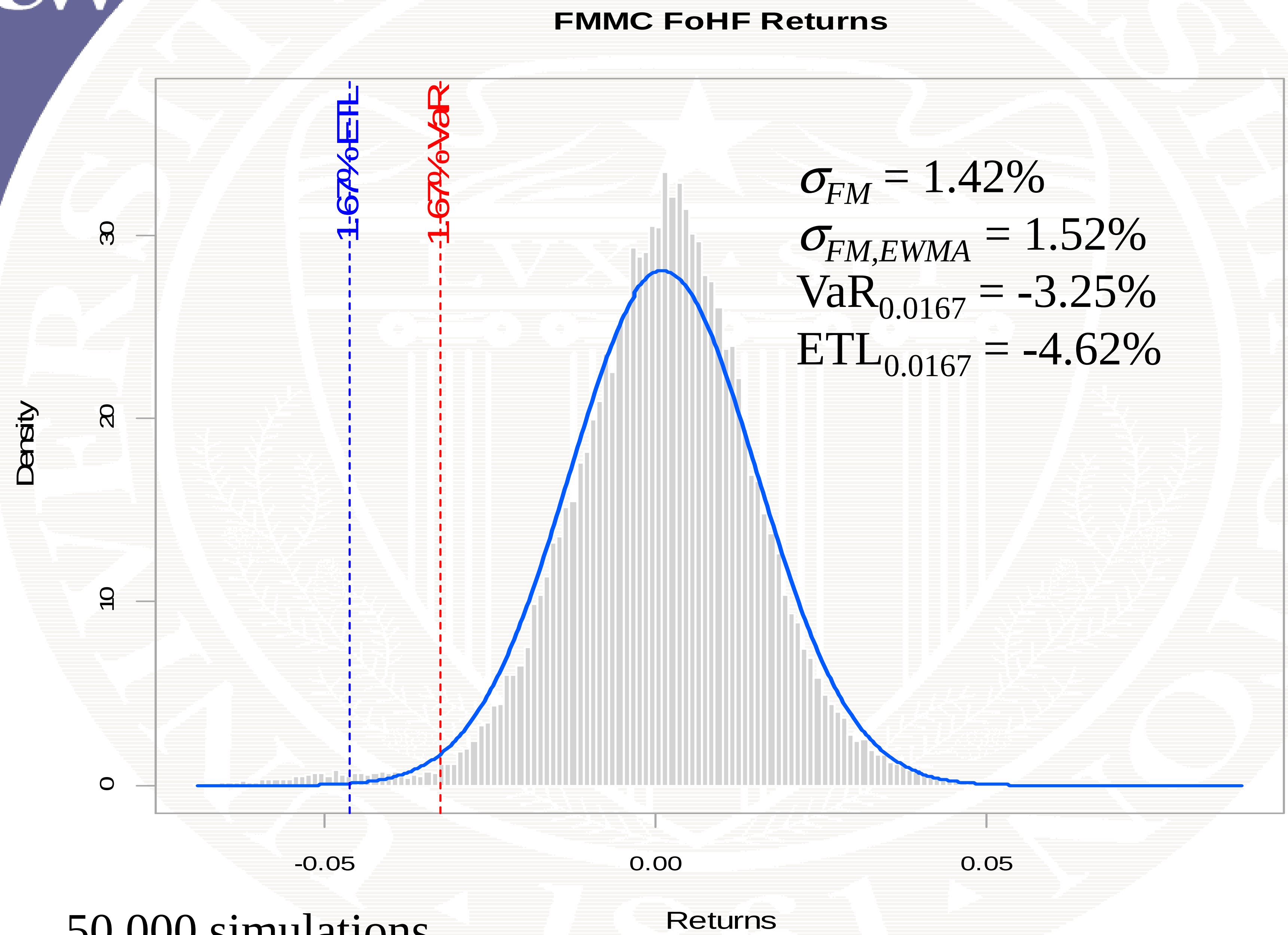
5% VaR



Fund marginal contribution to portfolio 5% ETL

Example FoHF Portfolio Analysis

- Equally weighted portfolio of 12 large hedge funds
- Strategy disciplines: 3 long-short equity (LS-E), 3 event driven multi-strat (EV-MS), 3 direction trading (DT), 3 relative value (RV)
- Factor universe: 52 potential risk factors
- R^2 of factor model for portfolio ≈ 75%, average R^2 of factor models for individual hedge funds ≈ 45%



σ_{FM} = 1.42%

$\sigma_{FM,EWMA}$ = 1.52%

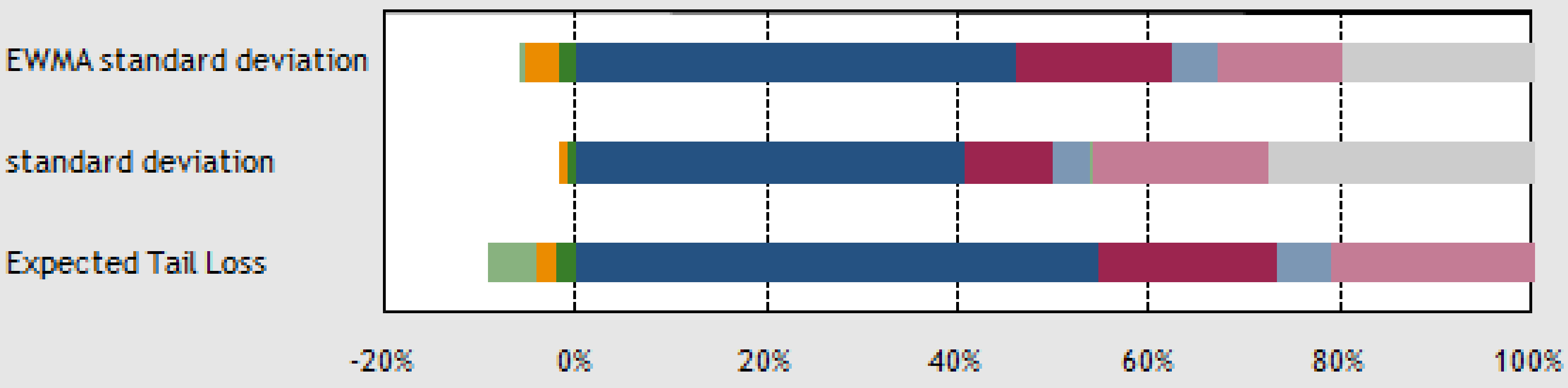
$VaR_{0.0167}$ = -3.25%

$ETL_{0.0167}$ = -4.62%

50,000 simulations

Factor Risk Contributions

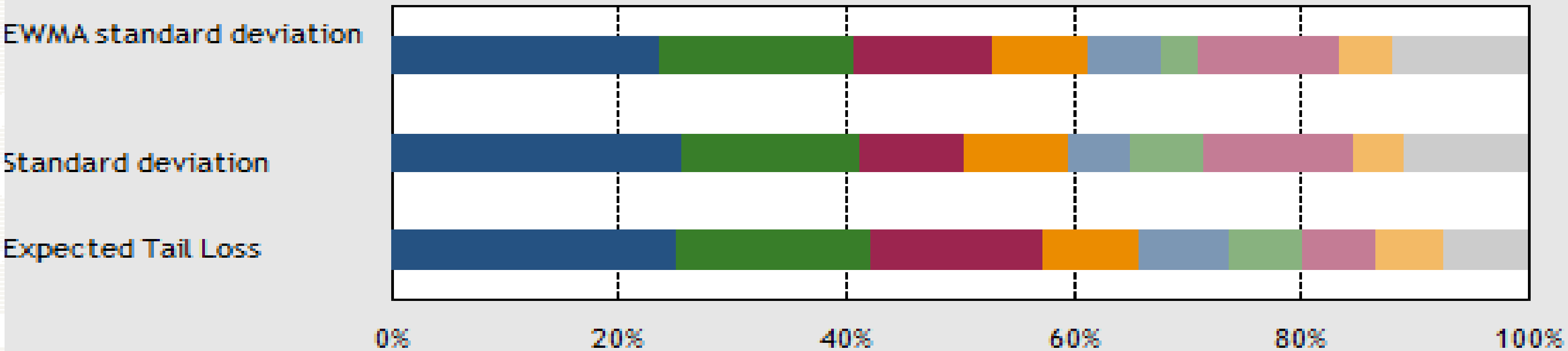
Factor Group % Contribution to Risk



Factor Group	EWMA Std Dev	Std Dev	Expected Tail Loss
Equity	46.3%	40.7%	54.8%
Rates	-1.8%	-0.8%	-2.0%
Credit	16.3%	9.4%	18.6%
Currency	-3.4%	-1.0%	-2.0%
Commodity	4.8%	3.8%	5.6%
Strategy - Observable	-0.5%	0.3%	-5.1%
Strategy - PCA	12.8%	18.3%	22.7%
Specific	25.5%	29.3%	7.3%

Hedge Fund Risk Contributions

Top 8 Programs by % Expected Tail Loss



Hedge Fund	Capital Allocation	EWMA SD Contribution	SD Contribution	ETL Contribution
Fund 3: LS-E	8.3%	23.5%	25.4%	24.9%
Fund 1: LS-E	8.3%	17.1%	15.7%	17.1%
Fund 4: EV-MS	8.3%	12.1%	9.0%	15.1%
Fund 9: DT-M	8.3%	8.5%	9.2%	8.4%
Fund 5: EV-MS	8.3%	6.3%	5.5%	8.1%
Fund 7: DT-M	8.3%	3.3%	6.4%	6.5%
Fund 6: EV-MS	8.3%	12.4%	13.3%	6.3%
Fund 12: RV-MS	8.3%	4.5%	4.4%	6.0%
Other	33.3%	12.1%	11.1%	7.6%

Hedge Fund Risk Contribution

Top 5 Programs by Lowest Marginal EWMA Standard Deviation		
Program	Capital Allocation	Maginal Short Risk
Fund 11: RV-R	8.3%	0.5%
Fund 2: LS-E	8.3%	0.8%
Fund 7: DT-M	8.3%	1.0%
Fund 8: DT-F	8.3%	1.1%
Fund 10: RV-MS	8.3%	1.2%
Top 5 Programs by Lowest Marginal Standard Deviation		
Program	Capital Allocation	Maginal Long Risk
Fund 2: LS-E	8.3%	0.5%
Fund 11: RV-R	8.3%	0.6%
Fund 10: RV-MS	8.3%	0.8%
Fund 8: DT-F	8.3%	1.2%
Fund 12: RV-MS	8.3%	1.2%
Top 5 Programs by Lowest Marginal Expected Tail Loss		
Program	Capital Allocation	Maginal ETL
Fund 8: DT-F	8.3%	-0.8%
Fund 2: LS-E	8.3%	1.5%
Fund 11: RV-R	8.3%	1.5%
Fund 10: RV-MS	8.3%	2.0%
Fund 12: RV-MS	8.3%	3.3%

Summary and Conclusions

- Factor models for asset returns are widely used in academic research and industry practice and are well suited to modeling hedge fund returns
- Tail risk measurement and management of hedge fund portfolios poses unique challenges that can be overcome using Factor Model Monte Carlo methods